Econ 802

Final Exam

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Read each question carefully and try to use all of the information provided. All questions have equal weight. If something is unclear, please ask.

1. A firm has the linear production function $y = ax$ where $x = (x_1 .. x_n) \geq 0$ is a vector of inputs and $a = (a_1 .. a_n) > 0$ is a vector of technology parameters. Let $w = (w_1 .. w_n) > 0$ be the vector of input prices.

(a) The firm wants to minimize the cost of producing a given output $y > 0$. In the long run what rule does the firm follow in deciding whether to use a positive or zero level of each input? What is the firm's long run cost function $c(w,y)$? Hint: use $L = -wx + \lambda(ax - y) + \mu x$ where λ is the Lagrange multiplier for the output constraint and μ is a vector of Kuhn-Tucker multipliers for the constraints $x \geq 0$.

(b) In the short run input n is fixed at $x_n^0 > 0$ but all other inputs are variable. Assume $y > a_n x_n^0$. Is it possible for marginal cost to be the same in the short run and long run while average cost is higher in the short run than in the long run? Explain carefully.

2. Consider the utility function $u(x_1,x_2) = -(x_1 - x_2)^2$ where $(x_1,x_2) \geq 0$.

(a) Choose a consumption bundle x^0 for which $x_1^0 \neq x_2^0$ and draw a graph of the upper contour set $\{x \geq 0: u(x) \geq u(x^0)\}$. Are these preferences strongly monotonic? Weakly monotonic? Locally non-satiated? Explain briefly.

(b) Suppose you observe (p^t, m^t, x^t) for $t = 1 .. T$ where (i) all prices and incomes are strictly positive, (ii) each bundle x^t satisfies $p^t x^t = m^t$, and (iii) each bundle x^t solves the problem max $u(x)$ subject to $p^t x \leq m^t$. Your friend claims that the data were generated by maximizing the utility function $w(x) = \min \{x_1,x_2\}$. Could the data ever contradict your friend's hypothesis? Use a graph to explain why or why not.

3. A consumer has the expenditure function $e(p,u) = 2(up_1p_2)^{1/2}$.

(a) Derive the Hicksian demands $h_1(p,u)$ and $h_2(p,u)$. Graphically interpret the fact that the ratio $h_1(p,u)/h_2(p,u)$ depends on the price ratio but not the level of utility. Does this fact have any implications for observable behavior? Why or why not?

(b) Derive the Marshallian demands $x_1(p,m)$ and $x_2(p,m)$. Use the Slutsky equation to decompose the cross effect $\partial x_1(p,m)/\partial p_2$ into a substitution effect and an income effect. Briefly interpret your mathematical results in words.

4. There are n consumers with identical utility functions $u_i(x_i,y_i) = (-1/x_i) + y_i$ for i = 1 . . n and m firms with identical cost functions $c_j(z_j) = (z_j^2)/2$ for j = 1 . . m where z_j is firm j's output of the x good and $c_j(z_j)$ is the amount of the y good firm j uses as an input. The price of the x good is p and the price of the y good is fixed at one. Each consumer has an endowment $m_i > 0$ of the y good but no endowment of the x good. All quantities are non-negative. Entry by additional firms is not permitted.

(a) Draw a graph showing the demand and supply curves in the market for the x good (make sure the general shape of the curves is accurate). Solve algebraically for the equilibrium price and quantity of the x good as functions of n and m.

(b) Using a graph as in part (a), show what happens to consumer surplus, producer surplus, and total surplus if the government sets a minimum price p^0 for the x good that is higher than the equilibrium price p*. Show mathematically whether producer surplus rises, falls, or stays constant and interpret your result graphically.

5. Agent A has utility $u_A = \theta \ln x_{A1} + (1-\theta) \ln x_{A2}$ where $1/2 < \theta < 1$. Agent B has utility $u_B = x_{B1} + x_{B2}$. The aggregate supply of each good is $w_1 = w_2 = 1$.

(a) Each agent has the same individual endowment $w_A = w_B = (1/2, 1/2)$. Find the Walrasian equilibrium prices and the resulting allocation. Draw an Edgeworth box showing the endowment point, the budget line, and the equilibrium allocation, and explain your answer verbally.

(b) Now consider an economy with n_A people of type A and n_B people of type B, where $n_B \geq n_A$. A social planner wants to maximize $n_A u_A + n_B u_B$ where u_A and u_B are the utilities of a typical individual of each type. The aggregate resource constraints are $w_1 = w_2 = (n_A + n_B)/2$. What is the optimal allocation? What prices and individual endowments could be used to achieve this allocation as a Walrasian equilibrium?

6. Robinson Crusoe is endowed with w_K seeds and w_L hours of labor. He produces food according to $y = \min \{\alpha x_K, \beta x_L\}$ where $\alpha > 0$, $\beta > 0$, $x_K \geq 0$ is the input of seeds, and $x_L \geq 0$ is the input of labor. Assume $0 < \alpha w_K < \beta w_L$. Crusoe's utility is equal to the food he consumes (he gets no utility from leisure or left-over seeds). He has no endowment of food.

(a) Use a graph to show Crusoe's input endowment (w_K, w_L) and the isoquant passing through this endowment point. Briefly describe the optimal physical allocation of seeds, labor, and food.

(b) Construct a Walrasian equilibrium involving the prices (p_K, p_L, p_F) where the price of food is fixed at $p_F \equiv 1$. Assume Crusoe gets all of the profits from the firm (if any). Solve for the equilibrium input prices $(p_K^*, p_L^*) \geq 0$ and show that at these prices (i) Crusoe is maximizing utility subject to his budget constraint; (ii) the firm is maximizing profit subject to its technology constraint; and (iii) the markets clear for all three commodities (seeds, labor, and food).